12/7/17

"I've missed more than 900 shots. I've lost almost 300 games. 26 times I've been trusted to take the game winning shot and missed. I've failed over and over again in my life. And that is why I succeed."-Michael Jordan

HW: "Finding Absolute Max and Min" w/s #1, 3, 4

AIM: How do we find Extreme Values of functions?

Warm Up:

1) Identify the point(s) where there is a horizontal tangent to the function:

slope = 0
Derivative = 0

$$f(x) = 2x^3 - 3x^2 - 12x - 1$$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$(x-2)(x+1)$

$x = 2 \quad x = -1$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) - 1$

$f(2) = -21$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) - 1$

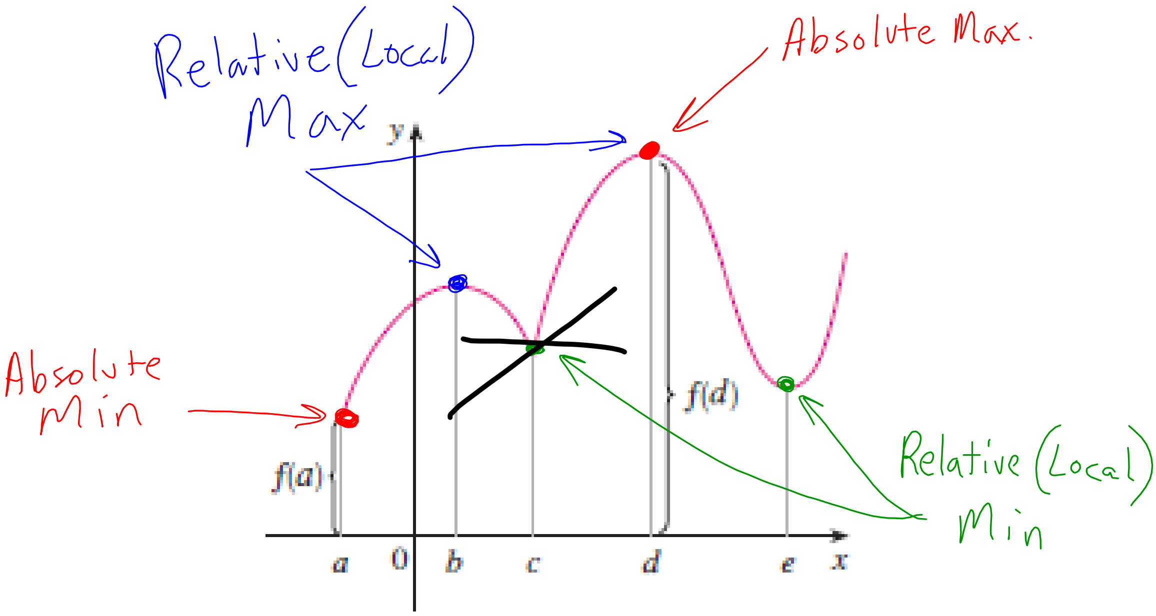
$f(-1) = 6$

Points w/ horizontal tangents
$(2, -21)$ and $(-1, 6)$

1 DEFINITION A function f has an **absolute maximum** (or **global maximum**) at c if $f(c) \geqslant f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called the **maximum value** of f on D. Similarly, f has an **absolute minimum** at c if $f(c) \leqslant f(x)$ for all x in D and the number $f(c)$ is called the **minimum value** of f on D. The maximum and minimum values of f are called the **extreme values** of f.

Highest Point = Absolute Maximum
Lowest Point = Absolute Minimum

2 DEFINITION A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geqslant f(x)$ when x is near c. [This means that $f(c) \geqslant f(x)$ for all x in some open interval containing c.] Similarly, f has a **local minimum** at c if $f(c) \leqslant f(x)$ when x is near c.



6 DEFINITION A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist. (fractions w/variable in denominator)

Find the critical numbers of $f(x) = 2x^3 - 3x^2 - 12x - 1$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6x^2 - 6x - 12$

$x = 2$

$x = -1$

(From Warm Up)

THE CLOSED INTERVAL METHOD To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

2. What is the maximum value of the following function on the interval [-2,3]?

$$f(x) = 2x^3 - 3x^2 - 12x - 1$$

x-values

$$f'(x) = 6x^2 - 6x - 12$$
$$0 = 6x^2 - 6x - 12$$
$$x = 2 \quad x = -1$$

Check the critical values and the endpoints of interval in the original function

$[-2,3]$ $x=2$ $x=-1$

$f(-2) = -5$

$f(-1) = 6$ ← Absolute max @ $\boxed{(-1, 6)}$

$f(2) = -21$

$f(3) = -10$

$\boxed{\text{Max value is } 6}$

3. What is the minimum value of the following function on the interval [-2,3]?

$$f(x) = 2x^3 - 3x^2 - 12x - 1$$

$\boxed{-21}$ from question #2

Min @ 2, -21

Examples) Find the absolute minimum and maximum values of the following functions

a. $f(x) = 3x^2 - 24x - 1 \quad [-1, 5]$

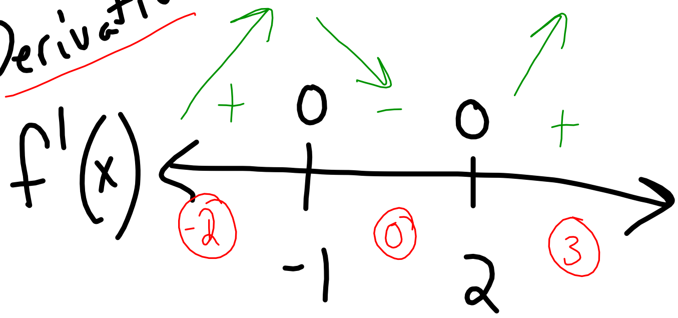
b. $f(x) = 6x^3 - 6x^4 + 5 \quad [-1, 2]$

4) When is $f(x) = 2x^3 - 3x^2 - 12x - 1$ increasing? Decreasing?

(Derivative > 0) (Derivative < 0)

⊛ Functions can change from increasing to decreasing (and vice versa) @ the critical values.

Derivative number line.



$f'(x) = 6x^2 - 6x - 12$

$f'(-2) = 24$

$f'(0) = -12$

$f(3) = 24$

Increasing: $(-\infty, -1) \cup (2, \infty)$

Decreasing: $(-1, 2)$